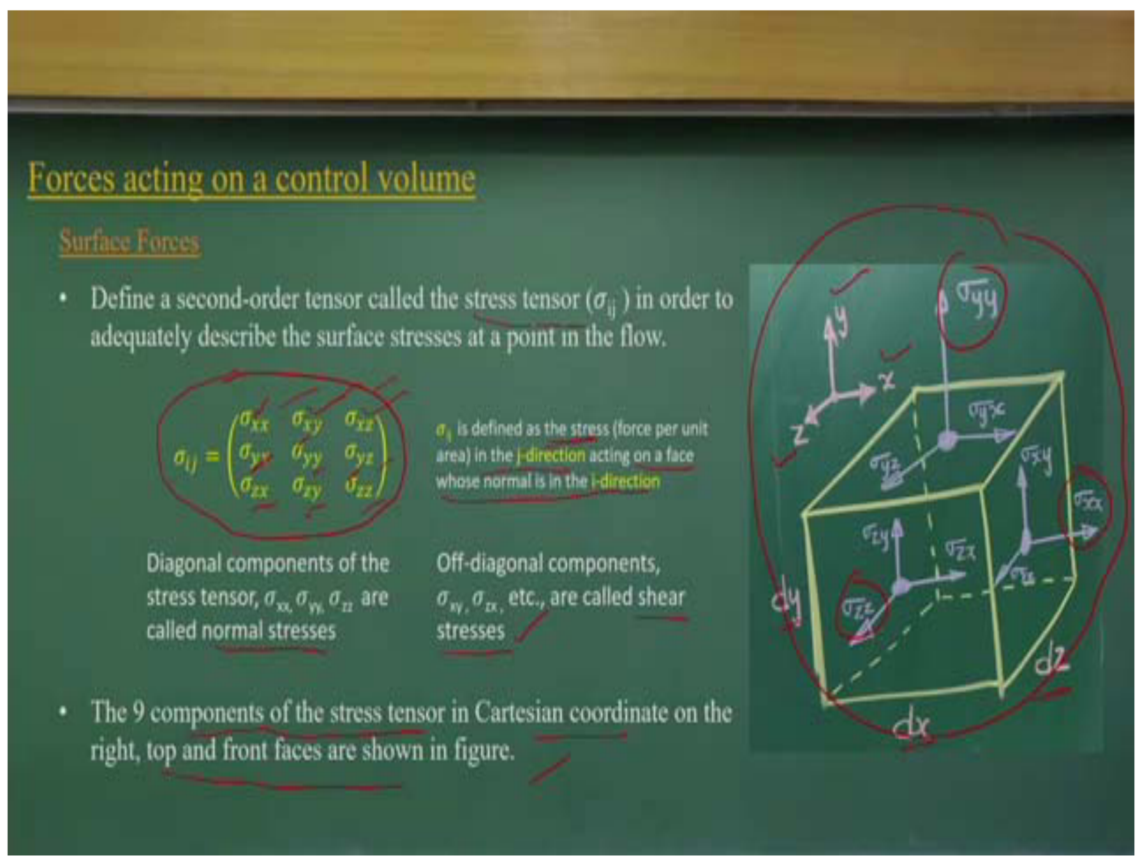

Now, if you come back to the surface forces, like for example, for tetrahedral structures like this where you have dx, dy, and dz components are there, and your axis is like this, x direction, y direction, and z direction. So, you can define the surface forces as a stress tensor. Stress means what, force per unit area. So, you can define as a stress tensor. So, this stress will have nine components. You could have this knowledge in solid mechanics.

I am just repeating it. There is not much difference between solid mechanics and fluid mechanics when you consider at stress level. So, we have stress tensors in order to describe all surface force components. That is what will have nine components which will have, as you know the subscript describes that. The stress in the z direction acting on the face whose normal is eight directions. This is similar notation to what we use in solid mechanics.

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

So, you have the stress tensor coordinate systems defining nine stress components. If you look at that, if you take this diagonal component which is the normal component to this surface like σ_{xx}, σ_{zz}, these are all normal components. That means these are compositions of the pressure force and the viscous force component. But the diagonal component what we have is σ_{xy}, σ_{xz}, and all, which is acting tangentially.

So, basically these are the viscous terms. So, over the surface we can define it, which is the shear stress components or the viscous stress component. So, these nine component of stress in Cartesian coordinate is defined in this surface. So, we can solve the problems considering the surface force defined as stress tensors and defining as normal stress and the shear stress component. The normal stress is a composition of pressure and viscous stresses, whereas shear stresses is only the viscous stress that we get.

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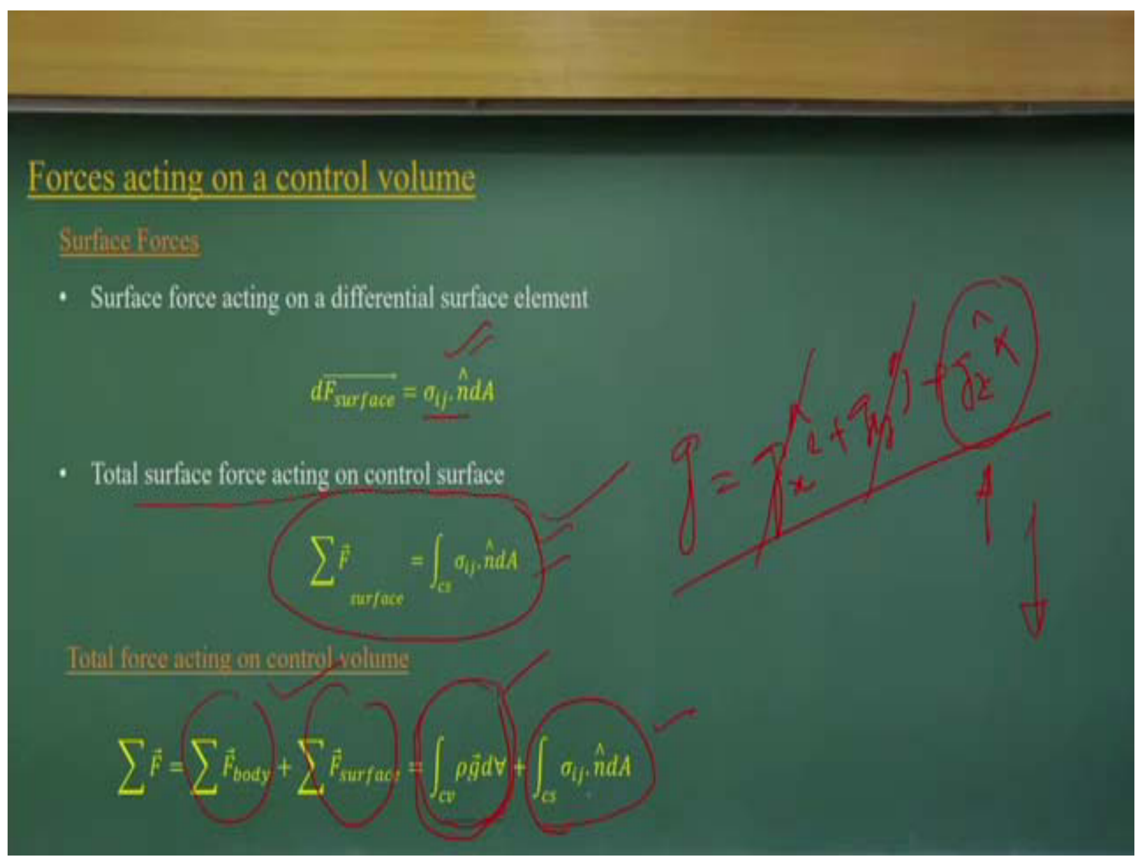

Surface force acting on a differential surface element

$$\overrightarrow{dF_{surface}} = \sigma_{ij}.\hat{n}dA$$

Now, if I have the stress component there and I have the normal vectors, if I resolve the force components, I will have the scalar product between the stress tensor and the n vectors, that is how we do it. And for the total surface area we do surface integrals to compute it, okay? Please do not be more worried about how we are having a scalar product of stress tensors and normal factors which will be coming to be again a second order vector components.

You try to get the mathematical point of view of that or different literatures nowadays available, you can understand the physics behind that, how mathematically we represent this stress tensors and the dot product or the scalar product of the stress and the normal vectors or any vector quantity. So, if it is that, you can integrate it to get all the stress components. So, total force acting on the control volume will have the body force component and surface force component.

Total surface force acting on control surface

$$\sum \vec{F}_{surface} = \int_{cs} \sigma_{ij}.\hat{n}dA$$

The body force component will have volume integrals of rho g dV, g is the vector quantity as we consider the g, acceleration due to gravity can have a vector commodity with three scalar components of g_x, g_y, g_z. That means what I am defining is $g = g_x\, I + g_y\, j + g_z\, k$. So, you can define like this. But many of the times we make alignments in such a way that only this k direction or negative direction and k is in upward direction.

We define the g that becomes a scalar quantity acting downwards. That is what happens. But if you are solving the problem where you do not know the direction of g vectors which is equal to $g_x\, I + g_y\, j + g_z\, k$, then you need to do the volume integrals to solve the problems and this is the surface integrals over this control surface to get what is the force acting on the surface.

Total force acting on control volume

$$\sum \vec{F} = \sum \vec{F}_{body} + \sum \vec{F}_{surface} = \int_{cv} \rho \vec{g} d\forall + \int_{cs} \sigma_{ij} . \hat{n} dA$$

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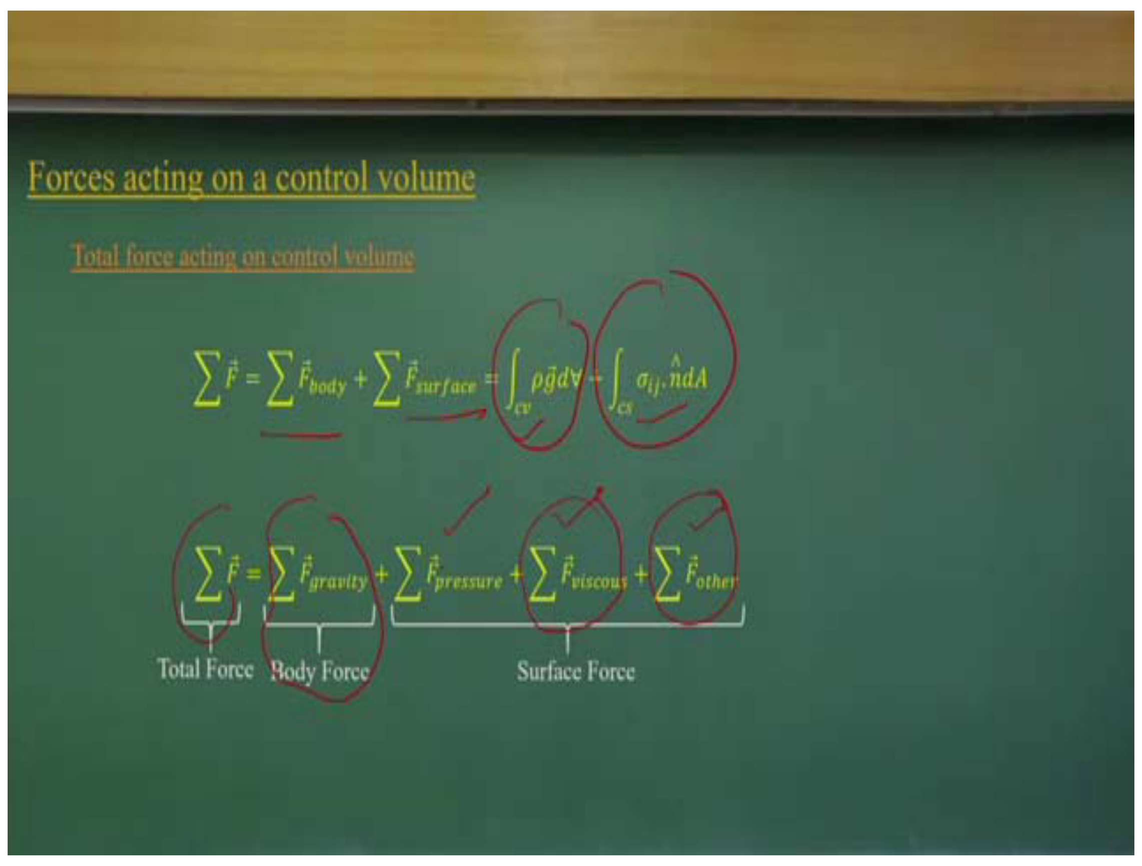

Then, we will go for simplification of this one. If you look at these things, it looks very complicated. We cannot apply simple example problems that we encounter as a civil engineer or mechanical engineers. What is the total force acting on the surface will be the body force and the surface force and this two integrals will tell me, one is volume integrals and other is surface integrals.

$$\sum \vec{F} = \sum \vec{F}_{body} + \sum \vec{F}_{surface} = \int_{cv} \rho \vec{g} d\forall + \int_{cs} \sigma_{ij} . \hat{n} dA$$

Now, if I divide the force components, that means, the total force will be one component from this body force, it is gravity force component. The surface force component we can resolve it into the force due to the pressure, force due to viscosity, force due to the other reactions. That means the reaction component of force. See, if I resolve this force component and if we can have a simplification like the cases where the viscous force is not dominated, then we can make it 0, or other forces not dominated we can make it 0.

$$\sum \vec{F} = \sum \vec{F}_{gravity} + \sum \vec{F}_{pressure} + \sum \vec{F}_{viscous} + \sum \vec{F}_{other}$$

That way, since we have resolved it or we separated the force components due to the pressure, due to the viscous, and separately for different types of problems we can simplify these equations and focus on the force components only due to the pressures or due to viscous or due to other components.

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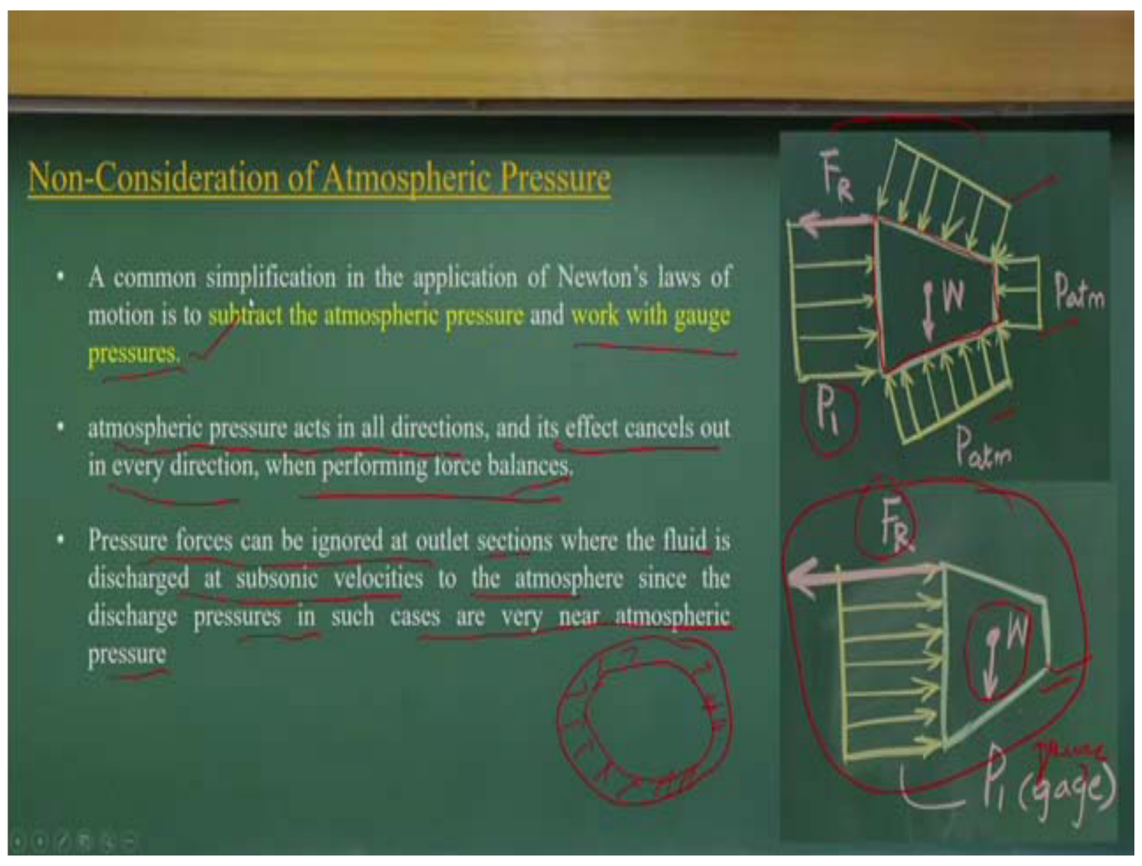

Now, let us consider very important component before applying this linear momentum equations. When you apply a linear momentum equations to a control volume, like I have the control surface like this, if you look at this control surface, in these three phases we have the pressure is atmospheric pressure. Here I have the absolute pressure, that means atmospheric pressure plus the gauge pressure.

If I look at any control volume I consider, the control surface always will have pressure equal to the atmospheric pressure. That means, if you can understand it, if I take a control volume, everywhere I will have atmospheric pressure, then the absolute pressure from some locations. So, if I consider the atmospheric pressure is acting throughout this control surface and do surface integral, if the pressure is having the direction, and if I do a surface integrals over this, it will be cancelled out and becomes 0.

So, considering that what we generally do it we nullify the atmospheric pressure component, because as you know when we integrate it the surface integrals of the atmospheric pressure for any control volume that becomes 0. So, we consider only the gauge pressure when you define the pressure diagrams for a control volume. For example, in this case, the atmospheric pressure components are given and this direction this pressure is equal to P_1 is the absolute pressure.

So, finally what we do is that we do not consider this pressure distribution as I said earlier. So, this part will be the gauge pressure, the difference between the absolute pressure and the atmospheric pressure. Then, we have the two force components working, one is the body force component and the other is the other reactions components. So, finally we use this control volume, a simplified control volume nullifying the atmospheric pressure distribution over the control surface.

We consider this control volume and the pressure diagram to solve the problems. That is, the atmospheric pressure acts in all the directions. Its effect is cancelled out in every direction when performing the force balancing, that is what I am saying, when you do surface integrals of this constant, pressure distribution in a close control volume, that is supposed to be 0 as it happens here. All these direction angles cancel out each other.

So, the pressure forces can be ignored at the outlet where the fluid is discharged at subsonic velocity to the atmosphere. Another assumption which is quite valid is that if you have subsonic flow, almost all times in civil engineering problems we get the subsonic flow, sonic or supersonic flow. In that case, the pressure force we generally we neglect in that state, as the discharge pressure in such cases is very near to the atmospheric pressure.

So, if you look and measure the pressure at that location when you have an outlet discharge, then that becomes the atmospheric pressure. So, we neglect that atmospheric pressure component if you have free outlet discharge. The point what I am trying to tell you is that we work with gauge pressure when you have defined the simplifications of control volume concept.

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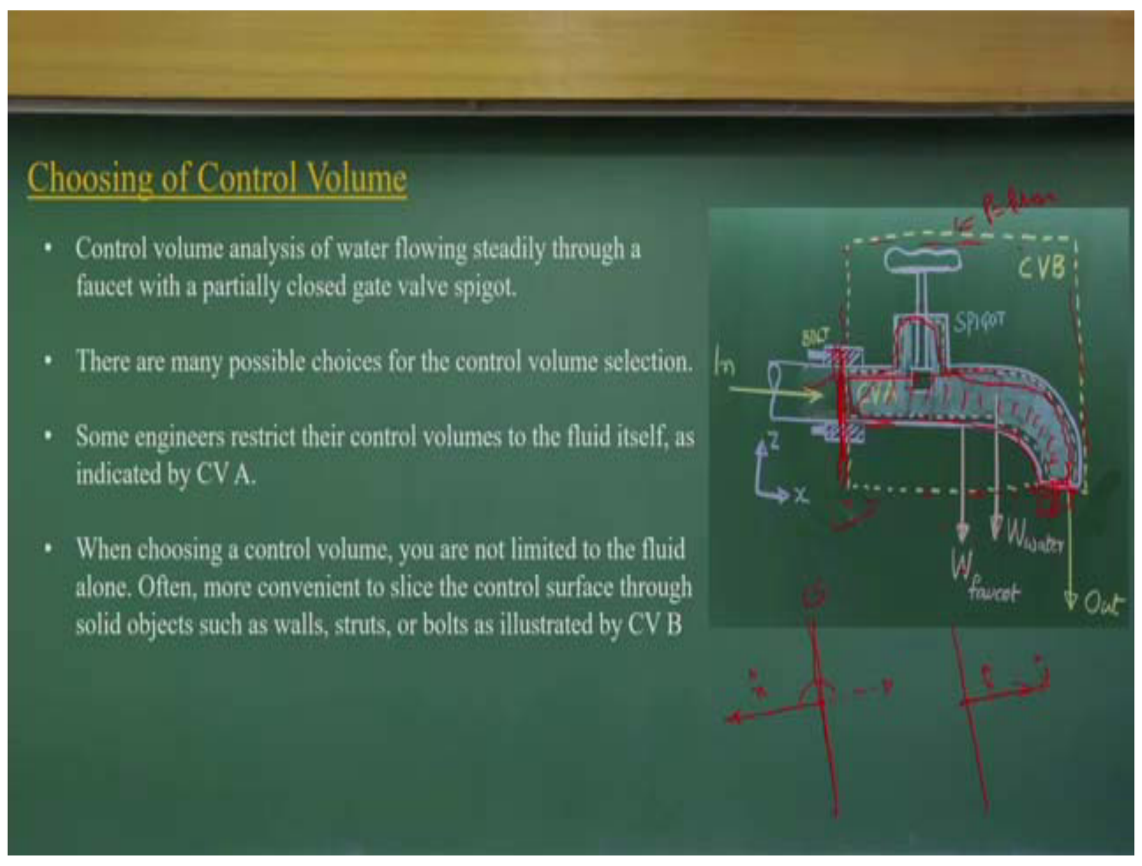

Now, another point is how to choose control volume, because that is what the art is, like you do free body diagrams in solving solid mechanics problems. Similar way, drawing the appropriate control volume is an art. That is what you have to learn by solving many problems using the control volume concept. Like, for example, what I need to do if do take this problem, okay?

So, that means there is water coming and there is a bolt holding the water outlet point and there is the spigot, and we have water coming out. I can have a control volume like as given here CV control volume, which is the wall surface touching only this water part if I consider the control volume. We do not know what is the stress acting over this surface. That is unknown to us.

Also, we do not know what is the pressure distribution at this point. So, when you draw this type of control volume, where over this control surface we do not know it, we cannot solve the problem. Instead of that, if I take a control volume like this, it is control volume B, if I look at this control volume, very clearly I know inflow, I know the weight component, one is water weight component and another is weight component of this tap or this spigot part.

Then, I also know this direction and q out from this. So, if I consider this control volume and over this P will be the P atmosphere. So, this problem becomes simple. If I just want to know how much of force is acting because of this flow orientation, so I can compute the force components to find out how much force is acting on this bolt. So, the point is that it is engineering skill or art to be developed by the students how to use the control volume concept. How to define the control volume for a flow system so that you can easily solve it.

And another thing what we do is that when you define the control volume, the control surface and the velocity should align to that. That means what I am telling you is that if my control surface is like this, if I have a normal vector to that, this is the normal vector, so V should have either 180 degrees in the same directions making the n vector and the v vector. If you do that, your scalar product becomes easy to do.
If $\theta = 0^\circ$, $\cos\theta = 1$,
if $\theta = 180^\circ$, $\cos\theta = -1$.

So, that way, it simplified this vector product. What you use in this thing is that the velocity and the normal vector they should have either 0° or 180°. That is what my point is. We can consider the control surface which will be with respect to velocity. We can take any shape, but when you take arbitrary shapes if your V and n is not having 0 or 180, finally you end up doing a scalar product, do surface integrals to solve the problem which is more time consuming and laborious, but the results will be the same.

So, what you have to look is how to define the control volume and the control surface in such a way that the target of your problems you have to solve it. That is art. That is what the art I will discuss in today's lecture. We will talk about the different types of control volume to use it. Next class onwards I will flow with how we should use appropriate control volume to solve the problems.
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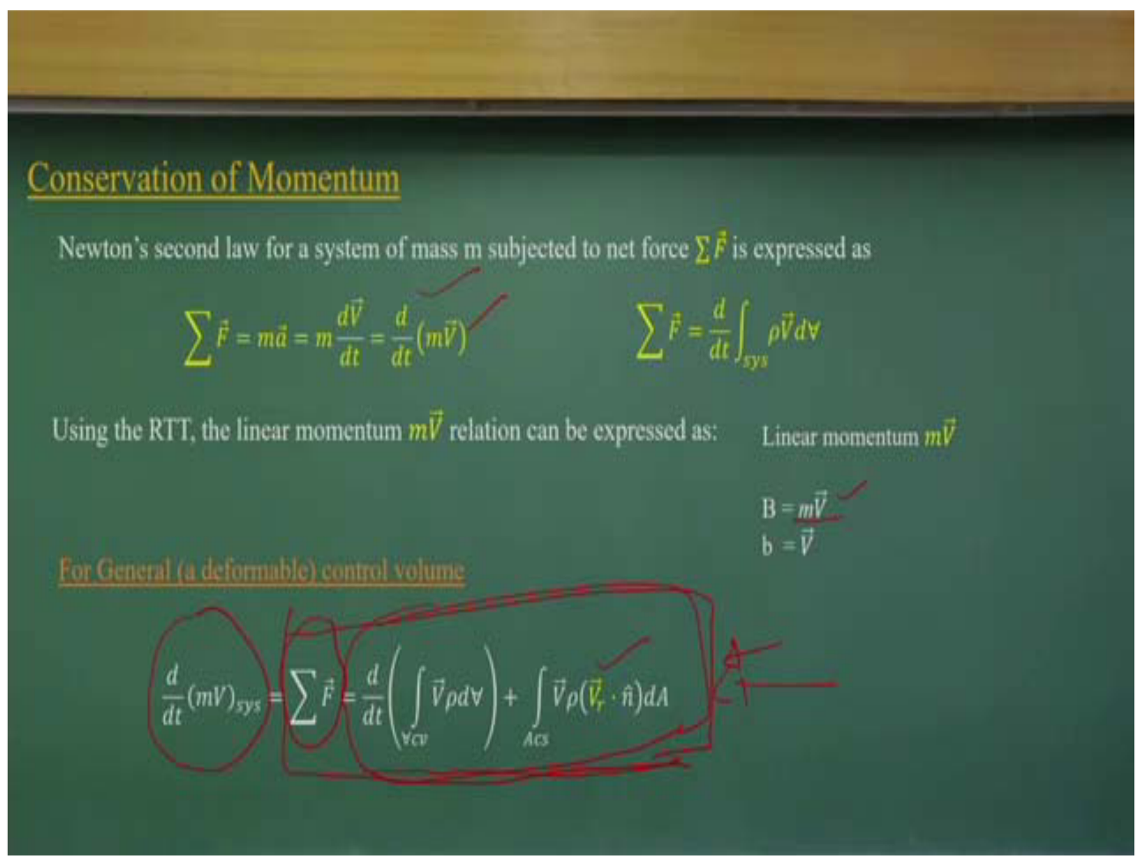

Now, coming back to applying the Reynolds transport theorem we have to write the linear momentum equations. At this systems level force is equal to mass into acceleration. That means, at the systems level force will be mass into acceleration as you know from basic solid mechanics. So, when I derive it at the systems level, the momentum flux, that is B, is my momentum flux. That means that should equal to net force acting on that.

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V})$$

$$\sum \vec{F} = \frac{d}{dt}\int_{sys} \rho \vec{V} d\forall$$

That is what is the system. And at the control volume levels, as B is equal to mass into momentum flux, the v, the intensive property becomes velocity vectors. If I apply with this, I will get this equation. So, this is the general equation for a control volume. We can have a V here. For moving control volumes we can use the relative velocity component and V is the velocity vector and V r is the relative velocity vector.

Using the RTT, the linear momentum $m\vec{V}$ relation can be expressed as:

Linear momentum $m\vec{V}$

B = $m\vec{V}$

b = $\vec{V}$

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}_r \cdot \hat{n})dA$$

So, this is the basic equation which is linear momentum equation in Reynolds transport theorem point of view.

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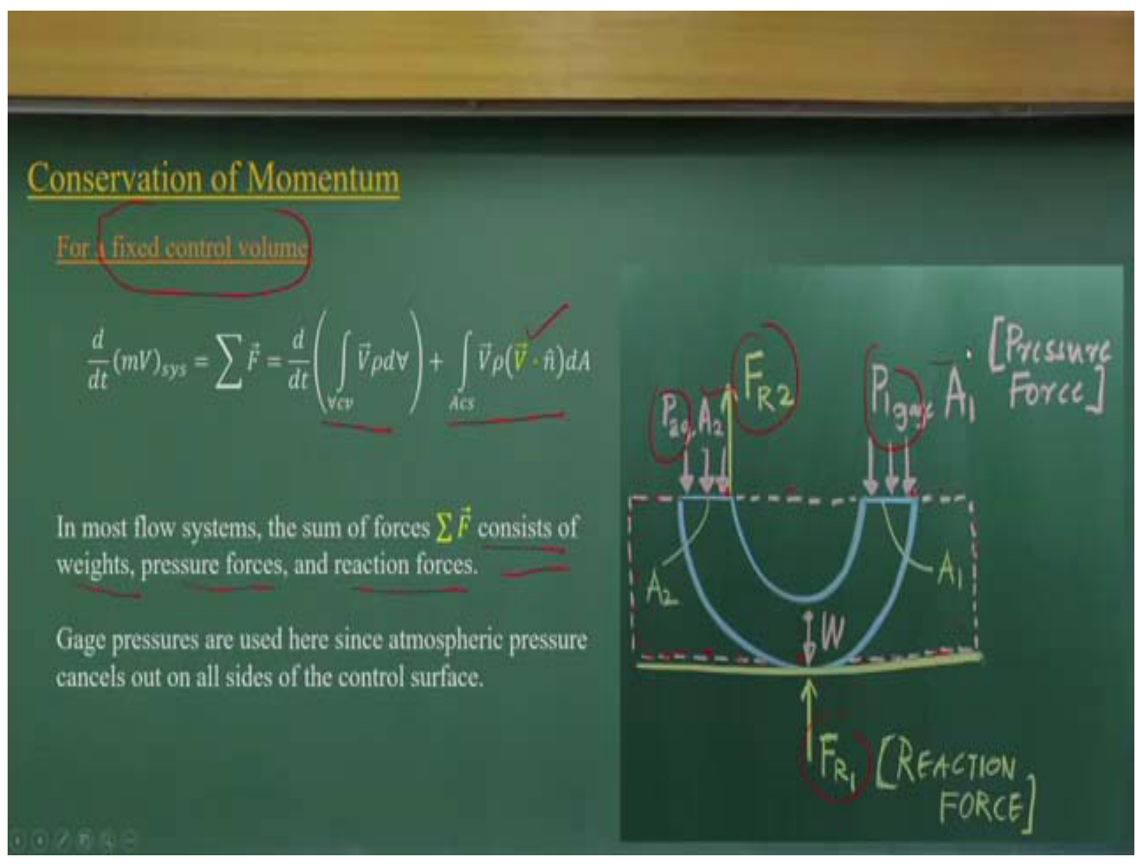

This equation will be simplified to solve the problems because, as of now, this problem we cannot solve because it has surface integrals and it is also having volume integrals. So, let us consider I have a fixed control volume, that means control volume is fixed. So, your V r becomes V in the case of this. So, you have volume integrals and surface integrals like this.

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

This is my control volume. The flow is coming from this.

Flow is also coming from this. Both the sides flow is coming in. And there the pressure is acting on P_1 and P_2 and A_1 and A_2 and there is a weight and there is reaction force acting on that. This is a A_1 and A_2 and this is pressure diagram. If it is that, this force can have, as I said earlier, we will have body force component which is the weight, the pressure forces, and the reaction forces at this point where we are getting the reaction forces.

Also another reaction force is attached to this. Here there are two contact points, at this and this. So, we can use gauge pressure. You need not do the integration over this. You can use a gauge at this two points to find out the pressure force component due to the flow.

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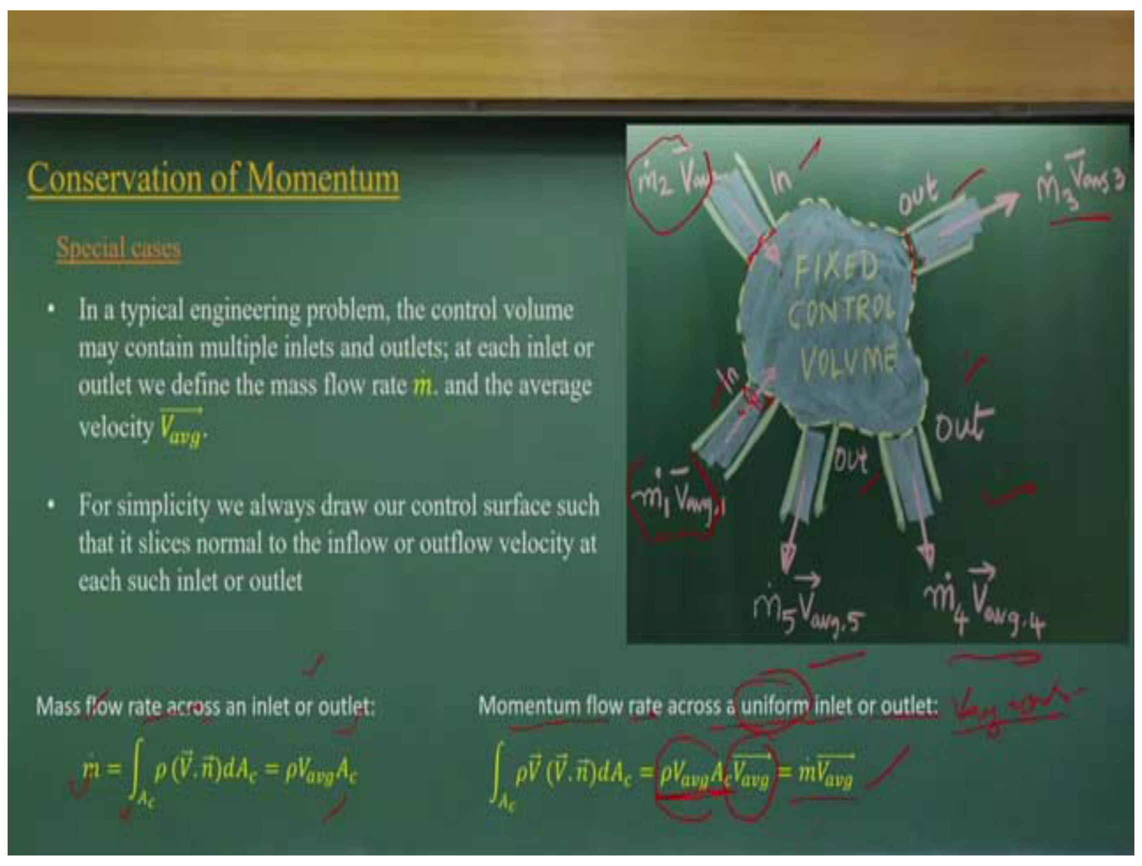

Now let us consider special cases. Like, many of the times, any industry if you look, there are network of pipes and connected to the tanks and all. Similar way, there will be a series of inflows, a series of outflow. If you have that type of concept, like, this is a fixed control volume, you have a series of inflows like 1 and 2 are the inflows, and 3, 4, and 5 are the outflows.

So, through this control surface if you look, we are talking about not only the mass flux coming into this control volume but we are talking about momentum flux. The momentum flux will be the mass flux, mass per unit time into the velocity, that is what will be the momentum flux. So, through this control volume we know this momentum flux. Through this control volume, what is the momentum flux coming into?

Through this control volume what is the momentum flux going out, and going out from this case, and this case, okay? So, basically, if you look, we can compute the momentum flux coming into this control volume and also going out. So, we have multiple inlets and outlet, then you can find out how much of net momentum flux is there in this control volume, just like we did for the mass flux in mass conservation equations.

Here, we are talking about net momentum flux passing through this control volume which will be equal to the net force acting on this control volume. We know force is equal to mass into accelerations. He same concept we are considering here as a change of the momentum flux within this control volume, that will be equal to the net force acting on this control volume.

So, now, if I consider velocity distributions are there, velocity variations are there, then, you know this mass flow rate, across this inlet and outlet we can have surface integrals to find out the mass flux. So,

Mass flow rate across an inlet or outlet:

$$\dot{m} = \int_{A_c} \rho\, (\vec{V}.\vec{n}) dA_c = \rho V_{avg} A_c$$

Similar way, we can find out momentum flow rate, momentum flux, if you assume it is a uniform inlet. That means your velocity is not changing. Momentum flow rate across a uniform inlet or outlet:

$$\int_{A_c} \rho \vec{V}\, (\vec{V}.\vec{n}) dA_c = \rho V_{avg} A_c \overrightarrow{V_{avg}} = \dot{m} \overrightarrow{V_{avg}}$$

V average is constant or the velocity variation is not there. That is what you call the uniform velocity. That condition does not vary. But some of the cases we can simplify that, okay? So, in that case, if in a surface velocity does not vary, so we can define it as mass flux, $\rho V_{avg} A_c$, that will be the mass flux, V average with the velocity, that will give the momentum flux.

So, here what we have considered is a uniform distributions of velocity. That is real fluid flow condition, we will have the correction factor for that.

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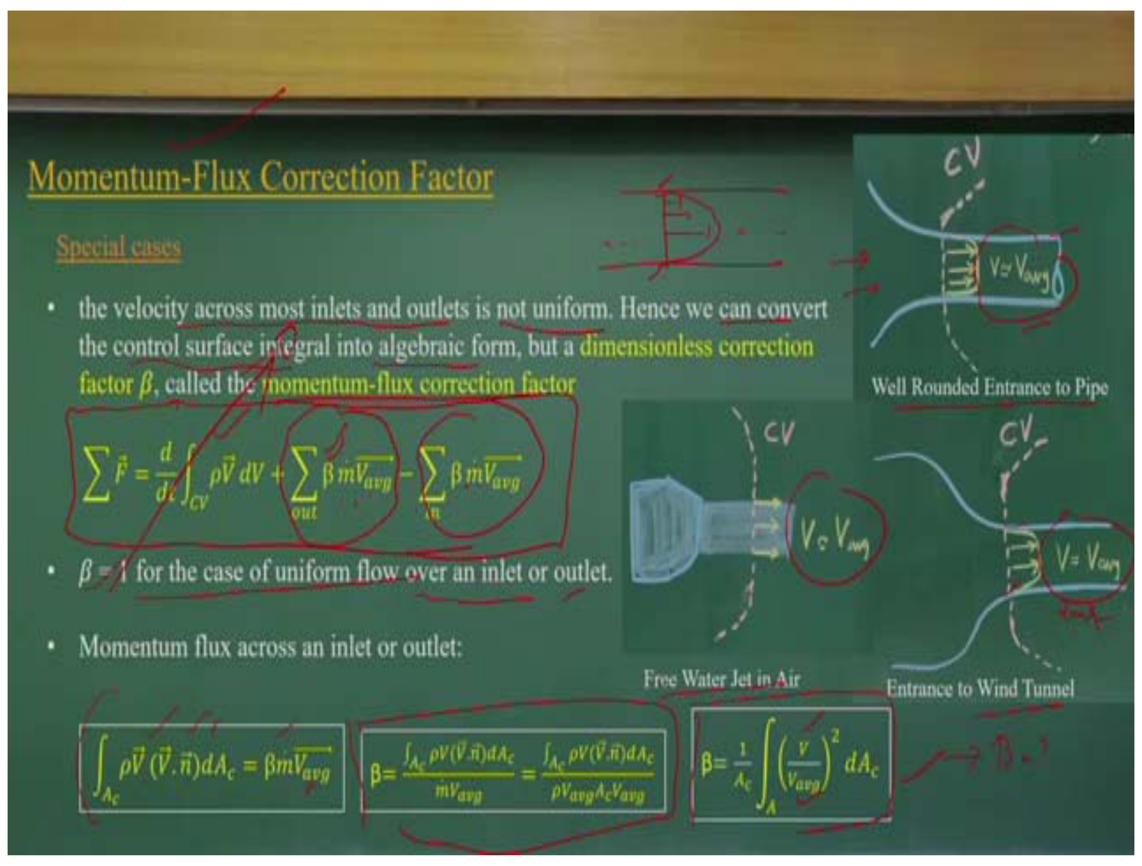

So, if you look it that way, now we can write the momentum equations in different forms considering the velocity distribution. That is the reason we introduce correction factors. That means, as we know it, the velocity distribution is not uniform for real fluid flow problems, computing this momentum flux using the average velocity, then what could be the correction factor for that.

That is what is called the momentum flux correction factor. Let me repeat that thing. You know for real life fluid flow systems, like for example, I have three examples here, okay? One of the examples is that there is well-rounded entrance to the pipe. The flow is coming from this, very well rounded, so you will see the velocity distributions will be there. At the wall it will be 0, then velocity distribution will be 0.

Similar way, you have a larger pipe section, wind tunnel section, then it is coming as test sections like this. This is the test section, okay, wind is coming through that. So, you know, over this control surface the velocity will be 0 here, then distributions, then coming back to that, or you have free water jet coming into air. So, you will have this. Most of the times what we have is absolute, where the fluid is turbulent.

No doubt you will have 0 velocity at the wall locations. The pipes or the test section is stationary at the rest conditions. The velocity distribution is more or less uniform except at the wall location. So, we can use average velocity concept for that. The V can be approximated as average velocity for all these cases. If not, let us have the pipe flow, the laminar pipe flow, so as you know you, velocity distribution will be 0 at the pipe contact location, the maximum velocity will be at the center.

So, there will be the parabolic distribution of velocity. In that case, as the velocity is not uniform, you need to have correction factors for computing the momentum flux. If we are computing momentum flux using average velocity concept, that is what is the correction factor. So, you try to understand it. So, if you are computing the momentum flux based on the average, then you should have a correction factor for that, okay?

Let us have this, the momentum-flux correction factor which will be the dimensional correction factor of beta where the velocity in most of the inlets are not uniform, we can convert this control surface integral to algebraic form. That is what it is here, okay? Because V average computations are easy and you know this mass flux in and out for each control volume, inlet and outlets, so you can compute the momentum flux.

And since you are using the average velocity to compute the momentum flux, you also consider the velocity distribution in terms of correction factors, that is what the β value is in terms of correction. So, finally, your surface integral part and the volume integral part will come like this. If it is a steady problem, this becomes 0. Again, you have a very simplified case.

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, dV + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

Sum of the force acting on that will be equal to the net momentum flux passing through this control surface, that is what will come, okay? So, let us have this for uniform flow. So, β will be equal to 1 and this thing if I do the integration of the velocities and the scalar products of V and n over a surface A_C, and that is what I am representing in average. So, β will come to be this part or with the simplification β will come this way.

If I know the velocity distribution, I know the average velocity. If I do this integration, I know this surface area, then I can compute what will be the beta value for a velocity distribution coming into a cross section. Like, I have a pipe of laminar flow, a pipe of turbulent flow, or it is an open channel flow it is connecting, but each case we know approximate velocity distribution. That means, for each case we know what is the β value.

$$\int_{A_c} \rho\vec{V}\,(\vec{V}.\vec{n})dA_c = \beta\dot{m}\overrightarrow{V_{avg}}$$

$$\beta = \frac{\int_{A_c} \rho V\,(\vec{V}.\vec{n})dA_c}{\dot{m}V_{avg}} = \frac{\int_{A_c} \rho V\,(\vec{V}.\vec{n})dA_c}{\rho V_{avg} A_c V_{avg}}$$

$$\beta = \frac{1}{A_c}\int_{A}\left(\frac{V}{V_{avg}}\right)^2 dA_c$$

So, if I know the β value, then we need do the integrations again. We use that beta value to convert from average velocity data to the momentum flux. So, that is the advantage to use the momentum-flux correction factor.

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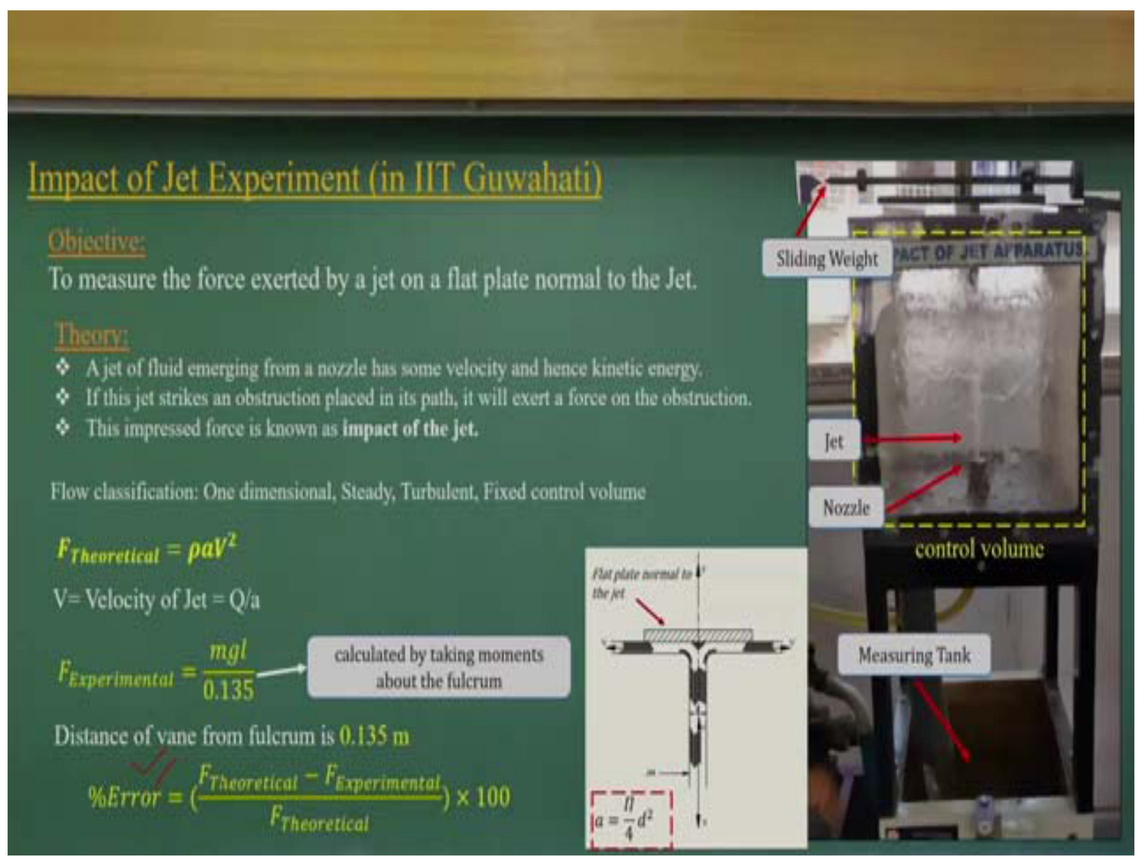

Now, before concluding today's lecture, let us see the figures, what is there, very simple experiment is impact of jet experiment. If you look at this jet, the water jet part is here and it is balanced by the weight here. So, we can know what is the velocity of impact if happening and that is how much of weight we are counterbalancing it. So, if you know it, theoretically, you know this is the control volume, you have water jet coming in and that water jet is going in these two directions.

If you look at this photograph and this simplified conceptual diagram, you can find out that. So, theoretically,

$$F_{Theoretical} = \rho a V^2$$

V= Velocity of Jet = Q/a

It is a momentum flux. ρ and V square will be the momentum flux that is impacting on that because in this direction the output momentum flux is 0. So, what is influx that is converted to the force component and you have experimental, then, definitely there is a deviation from the experiment and theoretical because any systems like this jet apparatus, it has some degree of loss.

$$F_{Experimental} = \frac{mgl}{0.135}$$

Distance of vane from fulcrum is 0.135 m

$$\%Error = (\frac{F_{Theoretical} - F_{Experimental}}{F_{Theoretical}}) \times 100$$

So, we cannot have exactly whatever the theoretical value. We will have deviations from that. That is why we compute the error. The deviations between the theoretical and experimental divided by theoretical, that is what gives us the error. So, this type of simple apparatus will be there in any engineering colleges, so you can see that how the force is acting because of jet impacting on that plate.

That is the condition. This is very simplified case. Here impact is normal to the plate. You can have many numerical examples where the plate can be inclined or the flow jet can have an incline and different conditions, we will also discuss in the next class.

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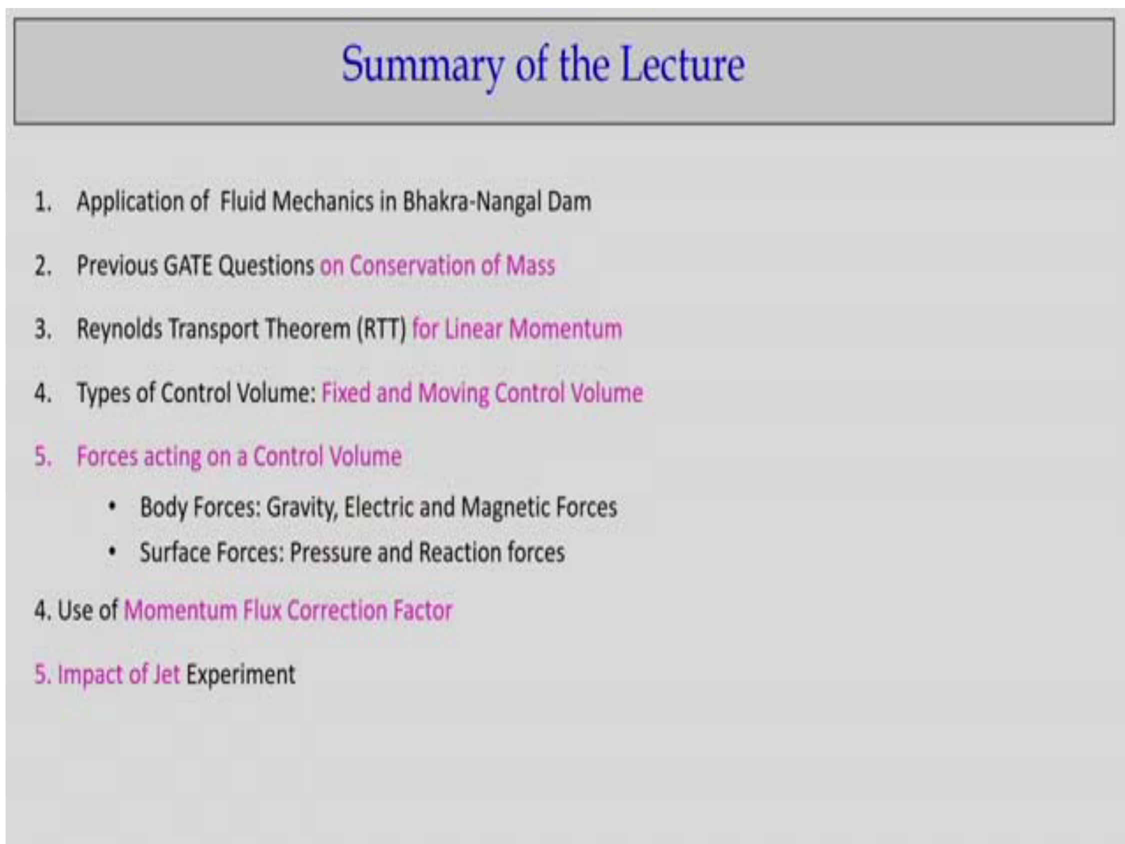

So, with this let me summarise today's class. We started with very interesting Bhakra and Nangal dam project. If you are that interested, you just get more data available, but I can say that because of that dam project we have changed the irrigation, hydropower generation, and water resource management in Himachal Pradesh and part of Uttar Pradesh and all. So, all because of the knowledge of fluid mechanics way back in 1950s and 1960s, that is how that is possible.

Now, we have more advanced way to understand the fluid mechanics. As I told earlier, we can solve many, many challenging problems apart from the standard problems. And we also discussed about the Reynolds transport theorem for linear momentum equations. The problems I have not solved, in the next class I will solve the problems and try to know how to know to use the control volume appropriately so that we can solve the problem with less timing and in proper way.

That is my point. So, we can solve problems or the exercise from any of the reference books. With this, let me conclude this lecture. Thank you.